Math 270: Differential Equations Day 11 Part 1

<u>Section 4.4</u>: The Method of Undetermined Coefficients (Part 2)

<u>Recall</u>

- In this section, we are trying to guess a particular solution y_p to the **non-homogeneous** 2^{nd} -order linear DEs w/constant coefficients ay'' + by' + cy = f(t)
- Our guess for y_p depends on what f(t) is

Recall

What would the form of our guess for y_p be in the following situations?

DE: $y'' + 3y' + 2y = 7t^4$

Recall

What would the form of our guess for y_p be in the following situations?

DE: $y'' + 3y' + 2y = 2e^{5t}$

Recall

What would the form of our guess for y_p be in the following situations?

DE: $y'' + 3y' + 2y = 2\cos 5t$

Recall

What would the form of our guess for y_p be in the following situations?

DE: $y'' + 3y' + 2y = t^3 e^{2t}$

Recall

What would the form of our guess for y_p be in the following situations?

DE: $y'' + 3y' + 2y = t^2 \cos 2t$

Discuss:

$$y'' + y' = 5$$

Discuss:

$$y'' - 6y' + 9y = e^{3t}$$

Method of Undetermined Coefficients

To find a particular solution to the differential equation

 $ay'' + by' + cy = Ct^m e^{rt},$

where m is a nonnegative integer, use the form

(14)
$$y_p(t) = t^s (A_m t^m + \cdots + A_1 t + A_0) e^{rt}$$

with

(i) s = 0 if *r* is not a root of the associated auxiliary equation; (ii) s = 1 if *r* is a simple root of the associated auxiliary equation; and (iii) s = 2 if *r* is a double root of the associated auxiliary equation.

Method of Undetermined Coefficients

To find a particular solution to the differential equation

$$ay'' + by' + cy = \begin{cases} Ct^m e^{\alpha t} \cos \beta t \\ \text{or} \\ Ct^m e^{\alpha t} \sin \beta t \end{cases}$$

for $\beta \neq 0$, use the form

(15)
$$y_p(t) = t^s (A_m t^m + \cdots + A_1 t + A_0) e^{\alpha t} \cos \beta t + t^s (B_m t^m + \cdots + B_1 t + B_0) e^{\alpha t} \sin \beta t ,$$

with

(iv) s = 0 if $\alpha + i\beta$ is not a root of the associated auxiliary equation; and (v) s = 1 if $\alpha + i\beta$ is a root of the associated auxiliary equation.

Example 5 Find the form for a particular solution to y'' + 2y' - 3y = f(t), where f(t) equals

(a) $7\cos 3t$ (b) $2te^t \sin t$ (c) $t^2 \cos \pi t$ (d) $5e^{-3t}$ (e) $3te^t$ (f) t^2e^t

Example 6 Find the form for a particular solution to y'' - 2y' + y = f(t), where f(t) equals

(a) $7\cos 3t$ (b) $2te^t \sin t$ (c) $t^2 \cos \pi t$ (d) $5e^{-3t}$ (e) $3te^t$ (f) t^2e^t

Example 7 Find the form of a particular solution to $y'' - 2y' + 2y = 5te^t \cos t$.